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A Study of Cleavability over g*p-Connected Spaces by Using Special Functions

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دراسة قابلية الانشطار على الفضاءات g*pالمترابطة بواسطة دوال خاصة

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Abstract

The purpose of this paper is to introduce a type of topological space called a p*g correlation space. The ability to fissile was also studied on this space, using some special functions such as the g * p-irresolute function and the m-pre open function. It was found that the g*p-connected space is a cleavable space when using some functions as follows, let X is a cleavable space and X be a g*p-connected space. Then Y is connected. Let X be a g*p-connected space is g*p-irresolute and M-pre-open cleavable over class P, Then Y is g*p-connected. Let X be pre-irresolute and M-pre-open cleavable over class P, H is a g*p-connected subset of Ythenf^(-1) (H) is a g*p-connected subset of X.

Keywords: *Open * groups, strongly open g * p function, irresolute * function, interconnected p * g space.*

الملخص

الغرض من هذه الورقة هو تقديم نوع من الفضاءات التبوبولوجية تسمىبفضاء الترابط g *p. كذلك تمت دراسة قابلية الانشطار على هذا الفضاء وذلك باستخدام بعض الدوال الخاصة مثل الدالةg*p-irresoluteوالدالة m-pre open . لقد وجد أن المساحة المتصلة بـ g * p هي مساحة قابلة للانقسام عند استخدام بعض الوظائف على النحو التالي ، دع X عبارة عن مساحة قابلة للكسر و X تكون مساحة متصلة بـ g * p. ثم يتم توصيل Y. لنفترض أن X عبارة عن مساحة متصلة بـ g * p تكون g * p غير ثابتة و M-pre-open قابلة للانقسام فوق الفئة P ، ثم Y تكون متصلة بـ g * p. دع X تكون متقطعة مسبقًا و M-pre-open قابلة للانقسام على الفئة P، H هي مجموعة فرعية متصلة بـ g * p من H) (Ythenf ^ (- 1) هي مجموعة فرعية متصلة بـ g * p من X.

الكلمات المفتاحية: مجموعات*p * g*المفتوحة ، دالة*p * g* المفتوحة بقوة ، دالةirresolute*p * g*، فضاء *p * g* المترابط.

1. Introduction

Levine (Levine, 1970) started the study of so-called g-closed sets in topological space (X, τ), where ,for any subset A of topology X is g- closed if the closure of A is contained in every open superset of A.

In 2002, Veera Kumar (2002) defined the concept of g*p-closed sets, g*p-continuity and g*p-irresolute maps. Patil et al., (2011) defined the concept of g*p-connected sets.

In 1985 Arhangl' Skii (1985) introduced different types of cleavability as following:

A topological space is said to be cleavable over a class of spaces $\mathcal{P}$ if for any subset of the space, there exists a continuous function belongs to $\mathcal{P}$ which is bijective function such that invers function of image of a subset A any subset of this space is the same set. In the last few years, many papers concerning cleavability were published especially the survey papers by Arhangl' Skii (1985).

Theorems of these concepts studied the relation between g*p-irresolute and M-pre open cleavable with g*p- connected space**.** Moreover, many examples are given to show g*p-separatedsubsets over g*p- connected space.

2. Preliminaries

In this paper, the spaces (X,τ) and (Y,σ) represent topological spaces on which no separation axioms are presumed unless explicitly stated. For a subset A of (X,τ), then $cl(A)$, $int(A)$ and A^c denoted the closure of A, the interior of A and the complement of A respectively in X.

Definition 2.1: A subset A of a topological space (X,τ) is called:

(i) $a\ semi-open\ set\ (Njastad, 1965)\ if\ A \subseteq cl(int(A))$.

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(ii) $an\ \alpha - open\ set\ (Popa, 1987) if A \subseteq int(cl(int(A)))$.

(iii) $a\ pre - open\ set\ (Mashhour) if\ A \subseteq int(cl(A))$.

The complement of preopen (resp. α-open, semi-open) set is called pre-closed (resp. α-closed, semi-closed).

Definition 2.2: For a subset A of a space (X, τ), then:

(i) The intersection of all pre-closed (Jeyachitra and Bageerathi, 2020), (El-Maghabi et al., 2012) sets including A is called the preclosure (resp. g*p-closure) of A and denoted by $pcl(A)$ (resp. g*p-$cl(A)$).

(ii) The union of preopen (El-Maghabi et al., 2012) (resp. g*p-open (Dontchev and Ganster, 1996)) sets included in A is called the pre-interior (resp. g*pinterior) of A and denoted by $pint(A)$ (resp. g*p-$int(A)$).

Definition 2.3: A subset A of a topological space (X, τ) is called:

(i) g$-closed$ (Levine, 1970) $if\ cl(A) \subseteq G\ whenever\ A \subseteq G\ and\ G\ is\ open\ in\ (X, \tau)$.

(ii) g*$p - closed\ (Veerakumar, 2002)\ if\ pcl(A) \subseteq G\ whenever\ A \subseteq G\ and\ G\ is\ g - open\ in\ (X, \tau)$.

The complement of a g-open (resp. g*$p - open$) set is g-closed (resp. g*$p - closed$).

Proposition 2.1: For a space (X, τ), we have:

Every preopen (resp. preclosed) set is g*p-open (resp. g*p-closed) (EL-Maghrabi and AL-Ahmadi, 2013).

Definition 2.4: A map $f : (X, \tau) \to (Y, \sigma)$ is called.

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(i) preopen (Mashhaur et al., 1982) if the image of each open set U of (X,τ) is preopen in (Y,σ).

(ii) M-preopen (1984) if the image of each preopen set U of (X,τ) is preopen in (Y,σ).

(iii) g*p-continuous [4] if the inverse image of each closed set U of (Y,σ) is g*p-closed in (X,τ).

(iv) g*p-closed (resp. g*p-open) (EL-Maghrabi and AL-Ahmadi, 2013) if the image of each closed (resp. open) set U of (X,τ) is g*p-closed (resp.g*p-open) in (Y,σ).

(v) strongly g*p-closed (resp. strongly g*p-open) (EL-Maghrabi and AL-Ahmadi, 2013) if the image of every g*p-closed (resp.g*p-open) set U of (X,τ) is g*p-closed (resp. g*p-open) in (Y,σ).

(vi) super g*p-closed (resp. super g*p-open) (EL-Maghrabi and AL-Ahmadi, 2013) if the image of each g*p-closed (resp.g*p-open) set U of (X,τ) is closed (resp. open) in (Y,σ).

(vii) g*p-irresolute (EL-Maghrabiet al., 2017) if the inverse image of each g*p-closed set U of (Y,σ) is g*p-closed in (X,τ).

(viii) strongly g*p-continuous (EL-Maghrabiet al., 2017) if the inverse image of each g*p-closed set U of (Y,σ) is closed in (X,τ).

Definition 2.5: A space X is said to be p-connected (Popa (1987) (resp. g*p-connected (Levine (1970))) if

$X \neq A \cup B$, where A,B are two non-empty disjoint preopen (resp. g*p-open) sets.

Definition 2.6: A subset B of a space (X,τ) is called p-connected (Popa, 1987) if it is p-connected as a subspace.

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3. *Generalized Preconnected Spaces

In this section, we introduce the notion of spaces called a g*p-connected space.Also, some of its properties is investigated.

Definition 3.1: Two non-empty subsets $A\,,B$ of a space (X,τ) are called g*p- separated if and only if $A\cap$g*$p-cl(B)=\phi\ and$ g*$p-cl(A)\ \cap B=\phi$.

Proposition 3.1: For a topological space (X,τ), the following statements are hold:

(i) Each separation set is g*p- separated,

(ii) Any two g*p- separated sets are always disjoint.

Proof:

(i) Let $A\,,B$ be two separated sets of (X,τ). So A ∩ cl(B) = ϕ and cl(A) ∩ B = ϕ, hence $A\cap$g*$p-cl(B)\subseteq A\cap cl(B)=\phi\,and$g*$p-cl(A)\ \cap B\subseteq cl(A)\ \cap B\ =\phi$.Then, $A\,,B$ are g*p- separated.

(ii) Assume that $A\,,B$ are g*p- separated sets. Then, $A\cap$g*$p-cl(B)=\phi\,and$ g*$p-cl(A)\ \cap B=\phi$. But, $A\subseteq$g*$pcl(A)\,and\,B\subseteq$g*$p-cl(B)$, so $A\cap\ B\subseteq A\cap$g*$p-cl(B)=\phi\,and\,A\cap\ B\subseteq$g*$p-cl(A)\ \cap B\ =\phi$. Therefore $A\,,B$ are disjoint.

The following example is explained that the converse of part (ii) of Proposition 5.1 is false.

Example 3.1: Let $X=\{1,2,3,4\}$ with a topology $\tau=\ \{X,\phi,\{1\},\{1,3\}\}$. Then two subsets $\{1,2\}\,,\{3,4\}$ of X are disjoint but not g*p -separated.

Theorem 3.1: Suppose that $A\,,B$ are two non-empty subsets of a space (X,τ). Then the following statements are holds:

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(i) If A, B are g*p-separated sets and H, G are non-empty sets of X such that $H \subseteq A$ and $G \subseteq B$, then H, G are also g*p-separated,

(ii) If $A \cap B = \phi$ and either both A, B are g*p-open or g*p-closed, then A, B are g*p-separated,

(iii) If both A, B are either g*p-open or g*p-closed sets and if $H = A \cap (X - B), G = B \cap (X - A)$, then H, G are g*p-separated.

Proof:

(i) Let A, B be two g*p-separated sets. Then $A \cap g{*}p - cl(B) = \phi$ and $g{*}p - cl(A) \cap B = \phi$. Since $H \subseteq A$and $G \subseteq B$, so $g{*}p - cl(H) \subseteq g{*}p - cl(A)$ and $g{*}p - cl(G) \subseteq g{*}p - cl(B)$. Then, $H \cap g{*}p - cl(G) = \phi$ and $G \cap g{*}p - cl(H) = \phi$. Therefore H, G are g*p-separated.

(ii) Suppose that A, B are two g*p-open sets of (X, τ). So $X - A, X - B$ are $g{*}p$ −closed sets. But $A \cap B = \phi$, then, $A \subseteq X - B$ which implies that $g{*}p - cl(A) \subseteq g{*}p - cl(X - B) = X - B$, therefore $g{*}p - cl(A) \cap B = \phi$. Also, we have $A \cap g{*}p - cl(B) = \phi$. Then A, B are $g{*}p$ −separated.

(iii) Assume that A, B are $g{*}p -$ open sets. Then $X - A, X - B$ are g*p- closed. Since $H \subseteq X - B$, $g{*}p - cl(H) \subseteq g{*}pcl(X - B) = X - B$ and hence $g{*}p - cl(H) \cap B = \phi$. Therefore, $g{*}p - cl(H) \cap G = \phi$. Also. We have $g{*}p - cl(G) \cap H = \phi$.

1. The proof is similar at A, B are g*p-closed sets. Then H, G are g*p-separated.

Theorem 3.2: If U, V are g*p-open sets of (X, τ) such that $A \subseteq U, B \subseteq V$ and $A \cap V = \phi$,

$B \cap U = \phi$, then A, B are non-empty g*p-separated sets of (X, τ).

Proof:

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Assume that U , V are g*p-open sets such that $A \subseteq U$, $B \subseteq V$, $A \cap V = \phi$ and $B \cap U = \phi$. Then $X - V$, $X - U$ are g*p-closed sets and hence $g^*p - cl(A) \subseteq g * p - cl(X - V) = X - V \subseteq X - B$ and $g^*p - cl(B) \subseteq g * p - cl(X - U) = X - U \subseteq X - A$. Therefore $g^*p - cl(A) \cap B = \phi$ and $g^*p - cl(B) \cap A = \phi$. Thus A , B are non-empty g*p-separated sets.

Definition 3.2: A topological space (X, τ) is called g*p-connected if $X \neq A \cup B$, where A , B are disjoint non-empty g*p-open sets.

Definition 3.3: A subset A of a space (X, τ) is g*p-connected if A is g*p-connected as a subspace of X.

Proposition 3.2: For a space (X, τ), every g*p-connected set is p-connected (resp. connected, g*p-connected). The converse of Proposition 3.2 is not true as is seen by the following examples.

Example 3.2: Let $X = \{1,2,3,4\}$ and $\tau = \{\phi, X, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$. Then a subset $\{2,3,4\}$ of X is p-connected (resp. connected) but it is not g*p-connected. Since$\{2,3,4\} = \{3\} \cup \{2,4\}$, where $\{3\}$, $\{2,4\}$ are g*p-separated.

Example 3.3: Let $X = \{1,2,3,4\}$ and $\tau = \{\phi, X, \{1\}, \{2\}, \{1,2\}\}$. Then a subset $\{2,3,4\}$ of X is g*p- connected but it is not g*p-connected. Since $\{2,3,4\} = \{2,3\} \cup \{4\}$, where $\{2,3\}$, $\{4\}$ are g*p-separated.

Theorem 3.3: For a space (X, τ), the following statements are equivalent.

(i) X is g*p-connected,

(ii) The only subsets of X which are both g*p-open and g*p-closed are the empty set ϕ and X.

(iii) Each g*p-continuous map of X into a discrete space Y with at least two points is

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2. a constant map.

Proof.

(i)⇒(ii). Assume that A is a subset of X which is both g*p-open and g*p-closed. Then $A^c$is also g*p-open and g*p-closed. Since X is the disjoint union of two non-empty g*p-open sets A and A^c , Hence one of them must be empty, that is, $A = \phi$ or $A = X$.

(ii)⇒(i). Suppose that X can be written as the disjoint union non-empty g*p-open subsets A , B of X. Then $A = B^c$, A is both g*p-closed and g*p-open sets. then by the assumption $A = \phi$ *or* X. Then X is g*p-connected.

(ii)⇒(iii). Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is a g*p-continuous map, where Y is a discrete space with at least two points. Then f^{-1} $(\{y\})$ is g*p-open and g*p-closed sets, for each $y \in Y$ and

$X = \cup \{f^{-1}(y) : y \in Y\}$. Then, by the assumption, $f^{-1}(\{y\}) = \varphi$ or X. If, $f^{-1}(\{y\}) = \varphi$, for all $y \in Y$, hence f will not be a map and there cannot exist more than one $y \in Y$ such that f^{-1} $(\{y\}) = X$. Then, there exists the only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and f^{-1} $(\{y_1\}) = \varphi$, where $y \neq y_1 \in Y$. Then f is a constant map.

(iii)⇒ (ii). Let S be both g*p-open and g*p-closed set of X. Assume that $S \neq \varphi$ and let.

$f: (X, \tau) \rightarrow (Y, \sigma)$ is a g*p-continuous map, defined by $f(S) = \{a\}$ *and* $f(S^c) = \{b\}$, where a, $b \in Y$ and $a \neq b$. Hence, f is a constant map. Therefore $S = X$.

4 -g*p-Connected - Cleavability

In 1985 Arhangl' Skii [2] introduced different types of cleavability as following:

A topological space X is said to be cleavable over a class of spaces $\mathcal{P}$ if for $A \subset X$ there exists a continuous function $f: X \rightarrow Y \in \mathcal{P}$ such that $f^{-1}f\ (A) = A$

, $f(X) = Y$.

In this section we study various types of cleavability over topological space called **g*p-connected** space by using especial continuous functions called M-pre-open (resp. M-pre-closed),g^*p -irresolute and pre-irresolute.

Theorem 4.1: Let X is cleavable space and X be a g*p-connected space. Then Y is connected.

Proof:

Suppose that Y is not a connected space. Then $Y = A \cup B$, where A, B are two non-empty disjoint open sets of Y. But, X is cleavable space, then there exists a surjective g*p-continuous map f, hence $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A), f^{-1}(B)$ are disjoint non-empty g*p-open sets of X, which is a contradiction with the fact that X is g*p-connected. So, Y is connected.

Theorem 4.2: Let X be a g*p-connected space is g*p-irresolute and M-pre-open cleavable over class $\mathcal{P}$, Then Y is g*p-connected .

Proof:

Suppose that Y is not a g*p-connected space. Then $Y = A \cup B$, where A, B are two non-empty disjoint g*p-open sets of Y. But, X is g*p-irresolute and M-pre-open cleavable over class $\mathcal{P}$, then $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$, $f^{-1}(B)$ are disjoint non-empty open sets of X which is a contradiction assumption that X is connected. then, Y is g*p-connected.

Theorem 4.3: Let X be pre-irresolute and M-pre-open cleavable over class $\mathcal{P}$, H is a g*p-connected subset of Ythen$f^{-1}(H)$ is a g*p-connected subset of X.

Proof:

Let $f^{-1}(H)$ be not a g*p-connected subset of X. Then there exist two disjoint non-empty

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g*p-open sets A , B, where f^{-1} (H) $=$ A ∪ B. Since, f is a bijective g*p-open map,

H $=$ $f(A)$ ∪ $f(B)$where $f(A)$, $f(B)$ are two disjoint non-empty g*p-open sets of Y , but by the assumption that H is a g*p-connected subset of Y . Then, f^{-1} (H) is a g*p-connected subset of X.

5- Conclusion

It was found that the g*p-connected space is a cleavable space when using some functions as following:

1) Let X is cleavable space and X be a g*p-connected space. Then Y is connected.
2) Let X be a g*p-connected space is g*p-irresolute and M-pre-open cleavable over class $\mathcal{P}$, Then Y is g*p-connected.
3) Let X be pre-irresolute and M-pre-open cleavable over class □, □ is a g*p-connected subset of □then□$^{-1}$ (□) is a g*p-connected subset of □.

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